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As

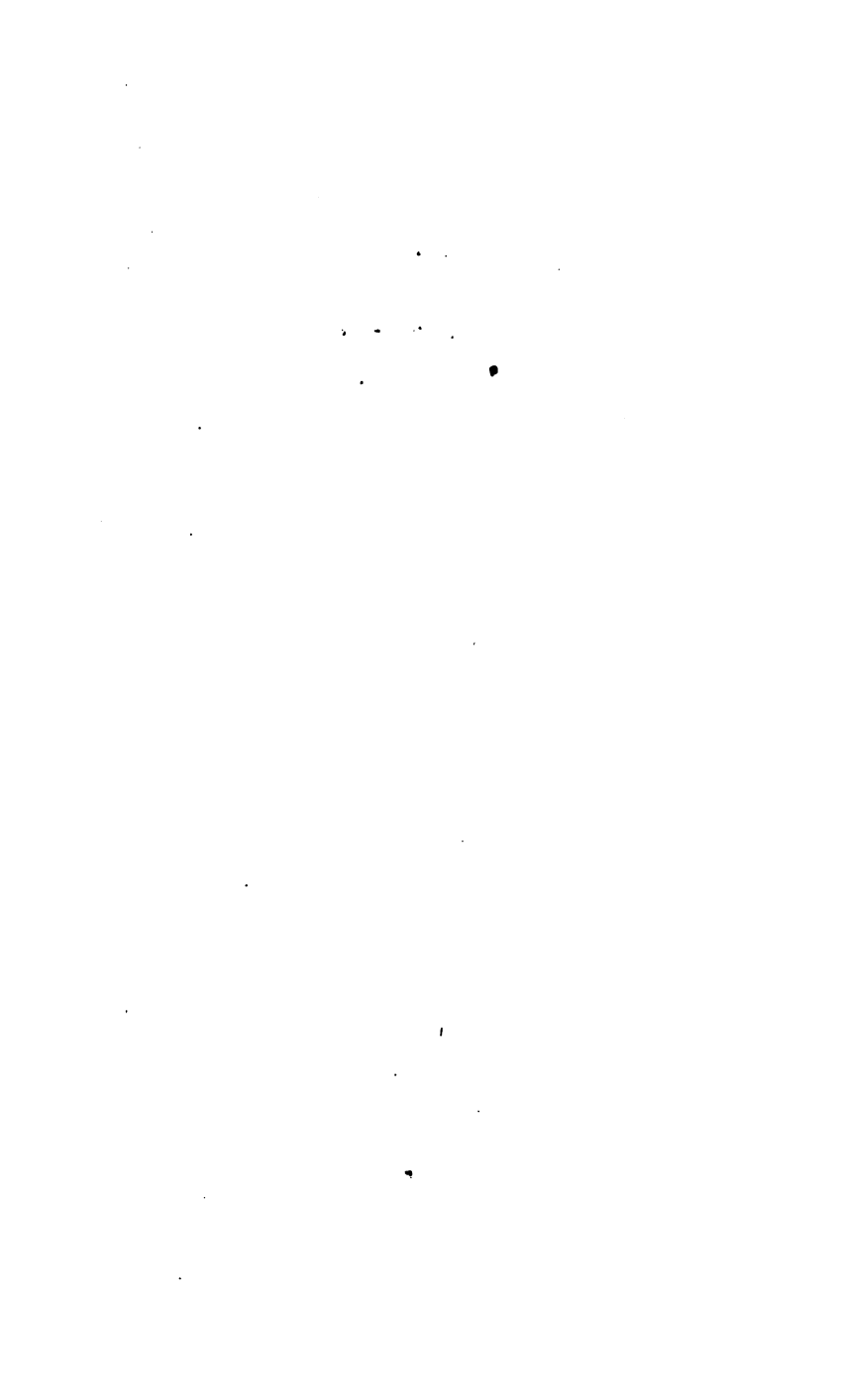
The image shows the front cover of a book. The cover is decorated with a marbled paper pattern. The pattern consists of large, irregular, dark brown or black shapes that resemble cells or stones, separated by thin, light-colored veins. Within these dark shapes are numerous smaller, circular or oval spots of a golden-brown or tan color. The overall effect is a dense, organic, and somewhat abstract design. In the top-left corner, there is a small, rectangular, light green paper label. On this label, the letters "As" are printed in a black, serif font. The book is positioned vertically, and the edges of the pages are visible on the right side.



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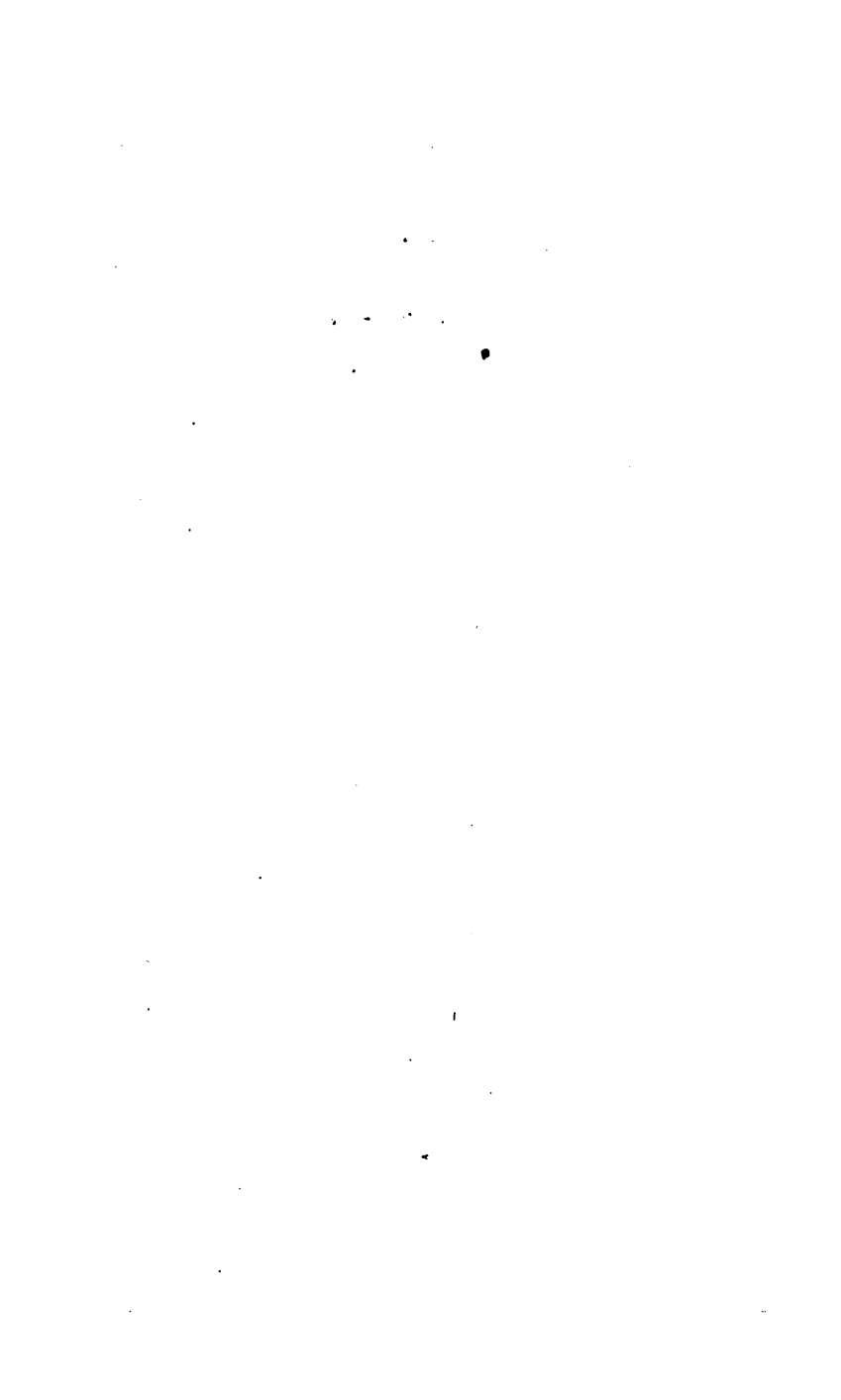




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AN EXPOSITION,
&c.

AN EXPOSITION
OF THE
NATURE, FORCE, ACTION, AND OTHER
PROPERTIES
OF
GRAVITATION ON THE PLANETS;
AND OF THE
EFFICIENT CAUSES OF THE
RELATIONS BETWEEN THEIR VELOCITIES, PERIODIC TIMES,
MEAN DISTANCES FROM THE SUN, AND THE
FORCES OF GRAVITATION.

BY THE AUTHOR OF
"A NEW ANALOGY FOR DETERMINING THE DISTANCES OF THE PLANETS FROM
THE SUN, AND OF THE SATELLITES FROM THEIR PRIMARIES,"
"A NEW INTRODUCTION TO THE MATHEMATICS," &c.

LONDON:
WHITTAKER AND CO. AVE MARIA LANE.
1842.

655.



GILBERT & RIVINGTON, Printers, St. John's Square, London.

ERRATUM

IN "AN EXPOSITION OF THE NATURE, FORCE, ACTION, AND OTHER
PROPERTIES OF GRAVITATION ON THE PLANETS."

Cancel the words "and the greater action an excess of $\frac{1}{2}$ of the distance C c" &c. in page 16, to the end of the sentence; and in lieu thereof, read as follows:—"and the action of that greater force generates a decrease of $\frac{1}{2}$ of the distance C c; but $\frac{1}{2}$ is the square root of $\frac{1}{2}$, wherefore the force is to the action inversely as the square root of $\frac{1}{2}$ to $\frac{1}{2}$. So that if B S represents the greater force, the ratio of B S to the resultant of the force and its action is compounded of the ratios of B S to $\sqrt{\frac{1}{2}}$, and of $\sqrt{\frac{1}{2}}$ to $\frac{1}{2}$; and if C S represents the resultant, or action of the force at C, then the ratio of B S to C S is compounded of the inverse ratios of B S to $C S^{\frac{1}{2}}$, and of $C S^{\frac{1}{2}}$ to C S. But the ratio of $C S^{\frac{1}{2}}$ to C S is inverse to that of the forces, wherefore the ratio of B S to $C S^{\frac{1}{2}}$ is compounded of the ratio of the forces and of the ratio of the greater force to its action; that is, because B S *does* represent the greater force at C, and C S *does* represent the action at C, B S is to $C S^{\frac{1}{2}}$ as the greater force at C is to its action."

cities and their distances, and between their velocities and periodic times. He showed that these analogies are conformable to the famous analogy discovered by Kepler in the

beginning of the 17th century, viz. that *the squares of the periodic times are as the cubes of the mean distances* ; by demonstrating it from his own two analogies, which he considered as an abundant confirmation of their truth. He was thence led to notice that similar analogies prevail among the satellites of Jupiter, and among the satellites of Saturn ; that is, that the satellites of each of those planets observe this relation as among themselves, but not as among the satellites of one of those primaries in common with those of the other, nor in common with the planets ; the cause of which diversity the Author was induced to ascribe to the different degrees of the force of gravitation of the Sun, of Jupiter, and of Saturn ; from thence he inferred and computed the ratios of the respective densities of the attracting bodies. But the Author did not in that short treatise attempt to trace these analogies to any pro-

ducing cause or causes, upon general principles, contenting himself with having established their existence, in point of fact.

Since the publication of that work, the Author has taken great pains to investigate the nature and properties of the gravitating and centrifugal *forces* (which are the two producing causes of the motions of the planets in their orbits); in which enquiry he proceeded upon established mechanical principles, and having in this manner computed the force of the Sun's gravitation upon each planet, he compared their respective gravitating forces thus found with their mean velocities, and he found the forces to be in the exact ratio of their velocities; and, consequently, inversely as the square-roots of the distances, and also inversely as the cube-roots of their periodic times. The success of this investigation emboldened him to engage in the difficult attempt to trace these

analogies up to those laws of motion, or to those mechanical principles, which the Author conceived must be their efficient, though latent, causes. The Author was encouraged to this arduous undertaking from the conviction that the analogies would be guides that he could depend upon in the desired discovery.

The only method of proceeding which the proposed discovery would admit of, was to form, *à priori*, a theory or hypothesis from which all these analogies might be demonstrated, or with which, at least, they would be consistent. This was a matter of imagination or of conjecture.

The Author's first conjecture (and he was so happy as not to have occasion to make a second) was that the *vis* or affection of gravitation proceeds from the Sun's whole hemisphere to the planets as a cone diminishing towards the planets, and terminat-

ing in points or vertices a little beyond them, According to this hypothesis the distances of the planets would be the altitudes of the cones ; and these altitudes, the resultants of the forces ; the ratio of which being inverse, it was to be inferred that the forces would be *inversely as the distances* or altitudes ; but this did not agree with our analogies, according to which the forces are *inversely as the square-roots of the distances*.

This discrepancy, however, did not induce the Author to abandon his conjecture as ill-founded. He considered that to a certain extent, though not altogether, the deductions from his hypothesis were conformable to the analogy ; and he considered that the want of their entire agreement was owing to some disturbing cause. This consideration set the Author upon the discovery of the disturbing cause.

He soon perceived that the *action* of gravi-

tation on the more distant of two planets was less oblique, and therefore greater than the action upon the nearer one, and that the ratio of this action was directly as the distances; this he considered the disturbing cause, and on investigating it he found it to be the case, and therefore that his hypothesis was conformable to the analogy.

In the following work the Author has demonstrated the analogies from general principles, with which his hypothesis is consistent; yet as his theories are pregnant with abundant matter for further speculations, he is aware that they ought not to be admitted without due investigation, lest they should prove to be a check, instead of a furtherance, to science.

London,
May, 1842.

AN EXPOSITION,

&c.

THE analogy discovered by Kepler in the beginning of the seventeenth century, viz., that the cubes of the mean distances of the planets from the Sun are as the squares of their periodic times, is found to be invariably consistent with observation, and is therefore so firmly established, as not to admit of any theory or hypothesis being advanced that purports to be at variance with it.

The author of these pages, taking this analogy as his basis, was induced to investigate the subject somewhat further, and in a short tract entitled, "A new Analogy for

determining the distances of the Planets from the Sun, and of the Satellites from their Primaries;" he has shown that *the mean velocities of the planets are inversely as the square-roots of their distances from the Sun, and also inversely as the cube-roots of their periodic times* ; which two analogies, as they confirm that of Kepler, are confirmed by his, and also confirm each other.

The Author was further led to enquire what could be the nature of the *force* and *action* of gravitation, so as to produce the analogies which are shown to subsist between the times, distances, and velocities of the planets.

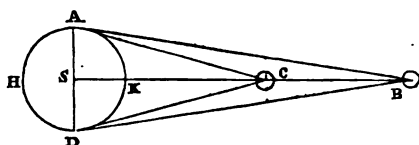
From these investigations (which have been made with the greatest diligence as well as caution) it appears that the Sun's gravitation acts upon the planets as if it emanated from his whole hemisphere, in the figure of a cone, of which his hemisphere is

the base, and of which the vertex is a point somewhat beyond the planet ; or taking the centre of the Planet as the vertex of the cone (which will be near enough for our present purpose), the distance of the Planet's centre from the centre of the Sun will be the altitude of the cone.

Adopting this hypothesis, therefore, we are to consider that the force of gravitation upon a planet at any distance from the Sun acts in combination with a centrifugal force, generating an uniform motion of the planet in a direction at right angles (or nearly so) with the direction of the force of gravitation, or of the centripetal force ; and that the centripetal force (as we shall presently show) is equal to the force of gravitation at the planet's actual distance from the Sun, so that the two forces balance each other, and by their composition keep the planet in its orbit. Hence we may consider the *distance*

of the planet from the Sun as the *resultant* of the force of gravitation. But inasmuch as this force diminishes as the distance increases, the ratio of the force of gravity to the distance is inverse, and consequently the *forces of gravitation*, taken independently of their action, *are inversely as the distances*.

But we are to take into the account that the *action* of gravitation on a more distant planet is *less oblique*, and consequently greater than the action of gravitation on a nearer one.



For let *s* represent the Sun, and *c* and *B* two planets. Considering the force of attraction to emanate from the Sun's hemisphere *A H D K*, the right line *B s* will be the result-

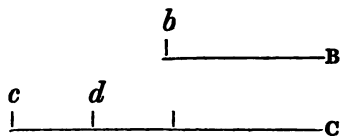
ant of the *action* of this force on the further planet B; and the right line cs will be the resultant of the action of the force on the nearer planet c . Hence the action of gravity at B, to that at c , is as BS to cs ; that is, it is directly as the distances.

Hence *the total force* of gravitation at B and c respectively (that is, *the force and its action together*) is compounded of the *inverse* ratio of the distances, and the direct ratio of the *action of gravitation*.

The ratio of the force of gravitation, taken apart from its action, is, as we have seen, inversely as the distances; that is, the force at c is to that at B, as BS to cs ; but the *action* of the force at c to that at B, is as cs to BS .

And since in this case the greater force has the less action, we are to see in what degree the less action operates upon the planet at c , compared with the operation of

the greater force at c . Let us suppose that the



force at c is double the force at B ; then (disregarding the *action* of the forces) in the time in which the force at B described the distance Bb , the force at c would describe cc , the double of Bb . But the *action* of the forces is such, that when the force at B has described Bb , the force at c (which would otherwise have described $2Bb$) has not described that distance by a space which is equal to half Bb ; that is, by a space equal to $cd = \frac{1}{4}$ th of cc ; that is, the greater force generates an excess of $\frac{1}{2}$ the distance cc , and the greater action an excess of $\frac{1}{4}$ of the distance cc ; but $\frac{1}{4}$ is the square root of $\frac{1}{2}$, wherefore the action at c to the force at c is as $cs^{\frac{1}{2}}$ to cs . The same will be the result whatever be the ratio of the forces.

Hence the *total force, that is, the force of gravity and its action at c, is as* $Bs \times Cs^{\frac{1}{2}}$.

And in the same manner it may be shown that *the total force at B, is as* $Cs \times Bs^{\frac{1}{2}}$.

Wherefore, if F represents the total force at B ;

D , the distance Bs ;

f , the total force at c ;

and d , the distance Cs ;

$$F : f :: d \times D^{\frac{1}{2}} : D \times d^{\frac{1}{2}};$$

and because of the equality of the products of extremes and means,

$$F \times D \times d^{\frac{1}{2}} = f \times d \times D^{\frac{1}{2}};$$

and, dividing by the last term of each equation,

$$\frac{F \times D}{D^{\frac{1}{2}}} = \frac{f \times d}{d^{\frac{1}{2}}}$$

and dividing by the denominators,

$$F \times D^{\frac{1}{2}} = f \times d^{\frac{1}{2}};$$

and making the fractions proportionals,

$$F : f :: d^{\frac{1}{2}} : D^{\frac{1}{2}};$$

that is, *the total centripetal forces are inversely as the square roots of the mean distances.*

Moreover, as we have above noticed, and as we have demonstrated in “A new Analogy,” &c., the mean velocities of the planets are inversely as the square roots of the distances ; that is,

$$v : v :: d^{\frac{1}{2}} : D^{\frac{1}{2}} ;$$

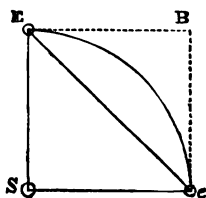
wherefore by equality of ratios,

$$v : v :: F : f ;$$

and because the velocities of the planets are known, being ascertained with great certainty by observation, this proportion affords another test for ascertaining the truth of our hypothesis.

In order to ascertain the total force of the Sun's gravitation upon a planet, or the centripetal force of a planet, e. g. of the Earth, we may consider the earth's orbit as being perfectly circular instead of being elliptical. In

this case the Earth, in one fourth of its revolution, would describe an arc of 90° ; that is, the arc Ee , in a quarter of a year, or 91.31 days.



Let E , be the position of the Earth at the beginning, and e , its position at the end, of that time; and let Es represent the direction of the action of gravity at the point E , and $EB (= Es)$ the direction of the centrifugal force, (or of the projecting force originally impressed upon the earth) at the point E .

It is obvious that when the earth arrives at the point e , it is in the same position as it would have been in, if, being at rest in the point E , it had been impelled by two equal constant forces acting at right angles to each other in the directions Es , EB ; in which case the Earth's path would have

been in the diagonal or chord ee . Comparing, therefore, the results, we may consider the simultaneous action of two constant equal forces, represented by the right lines EB , ES , as *equivalent* to the simultaneous action of the centrifugal force and of gravity on the Earth. Since ES and se ($= EB$) are radii of the earth's orbit, or its distance from the Sun, $= 95,000,000$ English miles, it is obvious that each of these constant forces, ES , and EB , would generate a velocity of 95,000,000 miles in 91.31 days, equal to 1,040,411 miles per diem.

The Earth would, by the simultaneous action of these constant forces move in the diagonal ee of the square $seBe$, with a velocity $= \sqrt{2(ES^2)}$; that is, (making radius, or R , $= ES$) $= \sqrt{2(R^2)}$; which, expressed in numbers, $= \sqrt{2(95,000,000^2)} = 134,350,000$ English miles in 91.31 days; which is equal to 1,470,261 miles per diem.

Now the velocity of the Earth, if her orbit were perfectly circular, would be

$$= \frac{2R \times 3.1415926}{365.25} = 1,634,223 \text{ miles per diem}$$

which exceeds what her velocity would be if she were propelled by the two constant forces above mentioned (viz. 1,470,261 miles per diem), by 164,062 miles per diem; which excess of velocity would be occasioned by her describing the arc Ee , by her gravity and centrifugal force, instead of describing the chord Ee by the two constant forces; that is, by passing through a greater space in the same time.

Since the force of gravity and the centrifugal force are equivalent, when acting simultaneously, to two equal constant forces acting simultaneously, we may consider the force of the Sun's gravitation upon the Earth, as equal to the Earth's centrifugal force; nay, the Earth's centrifugal force

being that which produces an uniform motion, can be no other than that force which, acting alone, would cause the earth to move from E to B in the right line EB, that is, to describe the distance or space of 95,000,000 miles in 91·31 days, or 1,040,411 miles per diem.

The properties of all the planets and of the satellites are the same in the above respects, if we consider their orbits as circular. Hence, if R represents the radius or distance of a planet from the Sun, or of a satellite from its primary; τ , the time or period of its revolution; c its centrifugal force; and g the force of the Sun's gravitation upon the Earth;

$$c = \frac{R}{\left(\frac{T}{4}\right)^2};$$

$$\text{also } g = c = \frac{R}{\left(\frac{T}{4}\right)^2};$$

which will give the quantities c and g in terms of their velocities in each of our days.

By which formulæ the force of the Sun's gravitation upon all the other planets is found to be nearly as follows :

Mercury . . .	1,681,818	miles English per diem.
Venus . . .	1,214,208	—
Mars . . .	837,209	—
Jupiter . . .	452,446	—
Saturn . . .	334,572	—
Georgian . . .	234,240	—

On comparing these forces (expressed in terms of their daily velocities) with the distances of the planets from the Sun, the forces are found by actual computation to be inversely as the square-roots of the distances, and consequently directly as the velocities, and thus our theory is established by fact.

Thus we have deduced this relation be-

tween the centripetal forces of the planets and their distances from the Sun, by a course of mathematical reasoning founded on the basis of Kepler's analogy; and we have determined upon other and different principles, the quanta of the centripetal forces, which are found to have that relation to the distances which our deductions from Kepler's analogy indicate. Hence we seem authorized to conclude that the analogy is established, according to the spirit of the fourth rule laid down by Sir Isaac Newton to be observed in deducing the laws by which the motions of the heavenly bodies are regulated, viz. "in experimental philosophy, propositions collected from the phenomena by induction are to be deemed (notwithstanding contrary hypotheses) either exactly or very nearly true, till other phenomena occur by which they may be rendered

either more accurate, or liable to exceptions."

This relation indeed is contrary to the received hypothesis, viz., that the centripetal forces of the planets are inversely as the *squares* of their distances from the Sun; whereas by our analogy the centripetal forces are inversely as the *square-roots* of their distances.

Thus in applying the *received* hypothesis to the force of the Earth's gravitation on the Moon, it is commonly said, that that force acting at the distance of the Moon from the Earth, is to the force of gravity at the Earth's surface inversely as the squares of those distances respectively from the centre of the Earth; and because the Moon's distance, 240,000 miles, is equal to about 60 semi-diameters of the Earth (4000 miles); therefore that the force of gravity at the Earth's surface is to its force at the

distance of the moon, as 60^2 to 1^2 , or as 3600 to 1.

By our analogy the ratio is as $60^{\frac{1}{2}}$ to $1^{\frac{1}{2}}$, that is, as 7.746 to 1.

But by our Formula,

$$g = \frac{R}{\left(\frac{T}{4}\right)}; = \frac{240,000}{7} = 34,285$$

miles per diem, as the force of the Earth's gravitation on the Moon, which is very much greater than the force of gravitation which the received hypothesis assigns to it.

In order to investigate this matter fully, it is necessary first to determine the force of the Earth's gravitation at her surface.

Let R , the Earth's semi-diameter = 4000 miles; T , the time of her rotation = 24 hours; and $g = c$; (that is, to be such a gravitating force as would exactly balance the centrifugal or rotatory force at

the Earth's surface, so that bodies at the Earth's surface, if without support, would not descend towards the centre); then would g be $= \frac{4000}{6} = 666\cdot6$ miles in 1 hour; or 16,000 miles per diem $= c$ also.

But the centrifugal force at the Earth's surface is not sufficient to balance the force of the Earth's own gravity, which causes all heavy bodies, if unsupported, to fall towards the centre with a velocity of 16 feet in the first second of time; and if we allow 16 feet per second as the obstruction of the *vis inertiae* of gravitating bodies, the excess of the Earth's force of gravity above her centrifugal force at her surface would be 32 feet per second, equal to 523·6 miles per day, which added to g would make 16523·6 miles per day as the actual force of the Earth's gravity at her surface.

Now, since the force of gravity at the Earth's

surface to that at the Moon, is by our analogy as 7·746 to 1, the force of the Earth's gravitation on the moon is $\frac{16523\cdot6}{7\cdot746} = 2133$ miles per diem ; which is only about $\frac{1}{16}$ part of 34,285, the actual centripetal force of the Moon.

If according to the received hypothesis the force of gravitation at the Moon were only $\frac{1}{3600}$ th part of what it is at the Earth's surface, the force of the Earth's gravitation on the Moon would be only $\frac{16523\cdot6}{3600} = 4\cdot6$ miles per diem, which is about $\frac{1}{7453}$ part of the Moon's actual centripetal force.

If our reasoning is well founded we are led to enquire by what means the Moon's actual centripetal force can be so great as 34,285 miles per diem, seeing that her gravitation towards the Earth is little more than at the rate of 2133 miles per diem. May there not be some other attractive power

between the Earth and the Moon besides gravitation, and greater than it?

Is this additional attraction magnetism? Can it be, that the Earth, as a magnet, attracts the Moon to her, in addition to their mutual attraction of gravitation? If some additional attraction, beside that of gravitation, is requisite to account for the appearances, or rather the realities of the phenomena of the motions of the Earth and the Moon, would the attraction of magnetism be adequate or fit for the purpose, or is there any other kind of attraction which we can propose in lieu of it?

It has been thought by some, that because the gravity of the Moon towards the Sun is greater than her gravity towards the Earth, the Moon at her conjunction ought necessarily to abandon the Earth, a consequence which would be inevitable if no other force acted on the Moon besides that of gravitation. In truth, the centripetal force of the

Moon to the Sun, is to her centripetal force to the Earth as thirty to one nearly; but both these centripetal forces are balanced by the Moon's centrifugal forces from the Sun and Earth respectively. Her revolution round the Sun is the *resultant* of her centripetal and centrifugal forces with regard to him; as her revolution round the Earth is the *resultant* of her centripetal and centrifugal forces with regard to the Earth. These resultants are disturbing forces to each other, and their composition occasions irregularities in the Moon's motions in her two orbits.

The Moon's continually keeping the same hemisphere turned towards the Earth, is consistent with the supposition that the magnetic influence subsists between the Earth and the Moon; for in that case the magnetic pole of attraction of the Moon would be continually turned towards the Earth, and her pole of repulsion from it.

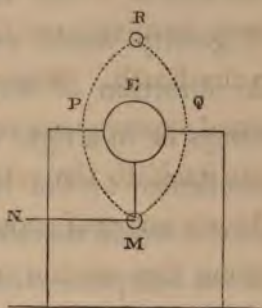
The time of the Moon's rotation corres-

ponds with her *synodic* revolution (that is, her period between one conjunction or opposition and another), and not with her *sidereal* revolution (that is, her period in passing through the perimeter of her orbit); the number of her synodic revolutions being one less in every year than that of her sidereal revolutions. For this coincidence the magnetic influence would be an adequate cause, and without such a cause the coincidence would appear fortuitous. Besides, to account for the Moon's continually turning the same hemisphere towards the Earth, except through the magnetic or some such like influence, there must be a combination of fortuitous circumstances, viz., a motion in such a direction as to countervail those new appearances of her surface which the Moon would present to the Earth if she had no rotation at all. It is obvious, that if the Moon's rotation were in the opposite direction to what it is, or if her axis were at right

angles to its present direction, she would in every revolution exhibit her whole surface to the Earth. Moreover, consistently with actual appearances, her axis must accommodate itself to the retrograde motion of the Moon's nodes of about $19^{\circ} 19' 44''$ annually, taking this motion at a mean, and what is much more, must accommodate itself to the *irregularities* of this motion of the nodes *in antecedentia*, although the causes which produce these irregularities in the Moon's motion in her orbit could have no effect upon her rotation. Now rotatory motion being circular is therefore uniform, and an uniform rotatory motion would be continually at variance with the irregular motions of the Moon in its orbit.

If we may be permitted to carry this reasoning further, we might suggest an experiment which would put our hypothesis to the test, or at least exhibit it in another form.

Let E represent a very powerful magnet, suspended by two horizontal strings, fixed to the tops of two upright pillars; and let a hollow ball of steel, M , be brought



within the influence of the magnet, and projected from M in the direction $M N$, at right angles to a plane conceived to pass through the string and the pillars; let the projecting force be conceived to be somewhat less than the magnetic force, and the magnetic force much greater (or 16 times greater) than the force of gravitation at M ; then, by the composition of the centripetal force $M E$, and the centrifugal force $M N$, the ball M , would describe the ellipse $M P R Q$; supposing those to be the only forces acting upon the ball; and the ellipse $M P R Q$, would be the resultant of the composition of those

two forces. With this resultant, the force of gravity would enter into composition, the direction of which latter force would always be in a right line from the ball perpendicular to the horizon. The force of gravity would therefore be a retarding force as the ball proceeded upwards from M to R through P , and would be an accelerating force as the ball proceeded downwards from R to M through Q ; and the resultant of the composition of this third force with the resultant of the two others, would be a depression of the altitude ER , and an elongation of the depth EM ; that is, it would lessen the excentricity of the orbit of the ball, which would continue to perform its revolutions round the magnet, in the same orbit which constituted its first revolution as the resultant of the composition of the three forces, centrifugal, centripetal, and gravity. For the acceleration of gravity, *as the ball descended* in its orbit, would be

equal to, and would therefore balance the retardation of gravity as the ball ascended ; and, consequently, the effect of gravity on the ball's motion would be expended in the first revolution. The only obstruction which the ball would afterwards meet with would be the atmospheric pressure, which would continually diminish (not the magnetic force, or the force of gravity, but) the centrifugal force, and consequently the mean distance of the ball from the magnet, until, finally, the centrifugal force being entirely destroyed by this continually acting obstruction, the ball would be drawn into contact with the magnet, and stop there.

This apparatus would be a kind of model, exhibiting similar appearances to those of the Moon and Earth. If a lighted candle were placed at some distance from it, nearly in the plane of the orbit, and if a spectator stood near the candle, with his eye on the same level with it, the ball in

each revolution would exhibit similar phases to those of the Moon in each of hers. The ball, when near its opposition, would exhibit the whole of its illuminated hemisphere (like the Moon when at full); when near its conjunction, the ball would present its dark side, as the Moon does hers at her change; and when the ball was in and near its quadratures, its bright parts would have the appearances of half-moons and crescents. Besides these appearances, the ball would, in each revolution, undergo an eclipse by the magnet, (viz. when in opposition,) and, when in conjunction, would perform a transit over the magnet's disc, to a spectator conveniently situated for observing these appearances.

But to return. Because $F : f :: v : v$; and because $v : v :: t^{\frac{1}{3}} : T^{\frac{1}{3}}$; it follows by equality of ratios that $F : f :: t^{\frac{1}{3}} : T^{\frac{1}{3}}$.

It remains therefore to discover and

assign the property or properties of the forces and motions from which this analogy springs.

In uniform motions the forces are as the velocities, and the forces and also the velocities are inversely as the times. Hence the squares of the forces in uniform motions are as the squares of the velocities, and inversely as the squares of the times; the times being understood to be those in which equal spaces are described.

If the orbits of the planets were circles, their motions would be uniform; and taking them to be so, it follows that since the velocities of any two planets are as the square roots of their distances from the Sun, the times in which those two planets would describe equal spaces in their orbits, are inversely as the square roots of *those* times. But when we speak of the *times* of the planets, we do not mean the times in which they describe equal spaces in their orbits, but the

periods in which they respectively describe their complete revolutions. Therefore, to distinguish the times in which equal spaces are described from the times of performing the revolutions, we shall call the latter *periods*, and use the word *times* in the former sense.

Since in uniform motions the times are inversely as the velocities, which as we have seen are as the forces; and because

$$v : v :: d^{\frac{1}{2}} : D^{\frac{1}{2}};$$

therefore $\tau : t :: D^{\frac{1}{2}} : d^{\frac{1}{2}},$

in which expression it is to be understood that D and d denote the distances from the Sun, and not the spaces described.

Now these proportionals denote the ratios of the velocities with which *equal spaces* would be described, and of the times in which equal spaces would be described. But the *spaces* described by the planets in an entire revolution are as their distances

from the Sun, so that the periods of the planets are compounded of the ratios of $D^{\frac{1}{2}}$ to $d^{\frac{1}{2}}$, and of D to d . Hence taking P p as denoting their periods,

$$P : p :: (D^{\frac{1}{2}} \times D) : (d^{\frac{1}{2}} \times d);$$

and multiplying extremes and means,

$$(P \times d^{\frac{1}{2}} \times d) = (p \times D^{\frac{1}{2}} \times D);$$

that is,

$$*(P \times d^{\frac{3}{2}}) = (p \times D^{\frac{3}{2}});$$

and squaring both sides,

$$P^2 \times d^3 = p^2 \times D^3;$$

and transforming the factors into proportionals,

$$P^2 : p^2 :: D^3 : d^3;$$

which is the expression of Kepler's analogy, and is therefore a demonstration of it.

Now we have assigned the causes from which we have demonstrated, first, that

$$P : p :: d^{\frac{1}{2}} : D^{\frac{1}{2}} \quad . \quad . \quad . \quad (A);$$

* For $d = d^{\frac{1}{2}} \times d^{\frac{1}{2}}$; wherefore $d^{\frac{1}{2}} \times d = d^{\frac{1}{2}} \times d^{\frac{1}{2}} \times d^{\frac{1}{2}} = (d^{\frac{1}{2}})^3 = d^{\frac{3}{2}}$.

and 2ndly, that

$$P^3 : p^3 :: D^3 : d^3; \quad . \quad . \quad (B).$$

Hence it is obvious that by squaring all the terms in the expression (A),

$$F^2 : f^2 :: d : D;$$

and extracting the cube-root of all the terms in the expression (B),

$$P^{\frac{2}{3}} : p^{\frac{2}{3}} :: D : d;$$

wherefore (by equality of ratios),

$$F^2 : f^2 :: p^{\frac{2}{3}} : P^{\frac{2}{3}};$$

and extracting the square-root of all the terms in this last expression;

$$F : f :: p^{\frac{1}{3}} : P^{\frac{1}{3}};$$

that is, *the forces are inversely as the cube-roots of the periodic times*; which is the remaining proposition that was required to be demonstrated.

THE END.





